(b) Use Taylor's theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$. (3,3)

8. (a) State and prove leibnitz's Theorem.

(b) If $y = \frac{\log x}{x}$, prove that

$$y_n = \frac{(-1)^n |\underline{n}}{x^{n+1}}\left[\log x - 1 \frac{-1}{2}\frac{-1}{3} \cdots\cdots \frac{-1}{n}\right].$$ (3,3)

(i) Printed Pages : 4]

(ii) Questions : 8]

Roll No. ....

Sub. Code : 0 0 4 4

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B.A./B.Sc. (General) 1st Semester Examination

1127

MATHEMATICS

(Calculus-I)

Paper : II

Time : 3 Hours] **[Max. Marks : 30**

Note :– (i) Attempt *five* questions, selecting at least *two* questions from each Unit.

(ii) Each question will carry 6 marks.

Unit–I

1. (a) Solve the inequation :

$$\frac{2}{x-2} < \frac{x+2}{x-2} < 2.$$

(b) State and prove Archimedian property. Using the property prove that the set of natural numbers N is not bounded above. (3,3)

2. (a) Show that $\lim_{x\to 0} \sin\frac{1}{x}$ does not exists.

(b) Evaluate :

$$\lim_{x\to 1/2} \frac{1}{x}\left[\frac{1}{x}\right], \text{ if exists.} \qquad (3,3)$$

3. (a) Use intermediate value theorem to show that equation $\sin x - x + 1 = 0$ has a real root.

(b) Evaluate :

$$\lim_{x\to 0} \frac{x - \sin x}{\tan^3 x}. \qquad (3,3)$$

4. (a) Evaluate :

$$\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right).$$

(b) Discuss the continuity of

$$f(x) = \begin{cases} \dfrac{|x|+x}{3} & , \; x \le 3 \\ \dfrac{2|x-3|}{x-3} & , \; x > 3 \end{cases} \text{ over R.} \qquad (3,3)$$

Unit–II

5. (a) Differentiate $y = x^{\sinh x} + x^{\cosh x}$ w.r.t. x.

(b) Let f be a real valued function defined in $[a, b]$ such that (i) f is continuous in $[a, b]$ (ii) f is differentiable in (a, b) (iii) $f(a) = f(b)$, then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$. (2,4)

6. (a) Prove that $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{x+1}{1-x}\right), -1 < x < 1,$ and then find its derivative.

(b) Use Cauchy's mean value theorem to evaluate

$$\lim_{x\to 1} \frac{\dfrac{\cos \pi x}{2}}{\dfrac{\log 1}{x}}. \qquad (3,3)$$

7. (a) Use mean value theorem to prove :

$$\frac{x}{1+x} < \log(1+x) < x \quad \text{for} \quad x > -1, x \ne 0.$$