Circuits

sec (5)

First Order Circuits

* RC Cuircuits

↳ $v_c(t) = V(\infty) + (V(0) - V(\infty))\, e^{-t/\tau}$. V

↳ $i_c(t) = C \dfrac{dv_c(t)}{dt}$. A

↳ $\tau = \underbrace{R_{equv}}_{\text{seen by the Capacitor}} \cdot C_{equv}$ "sec"

* RL Circuits

↳ $i_L(t) = I(\infty) + (I(0) - I(\infty))\, e^{-t/\tau}$. A

↳ $v_L(t) = L \dfrac{di_L(t)}{dt}$. V

↳ $\tau = \dfrac{L_{equv}}{\underbrace{R_{equv}}_{\text{seen by the inductor.}}}$ sec.

SHEET (2)

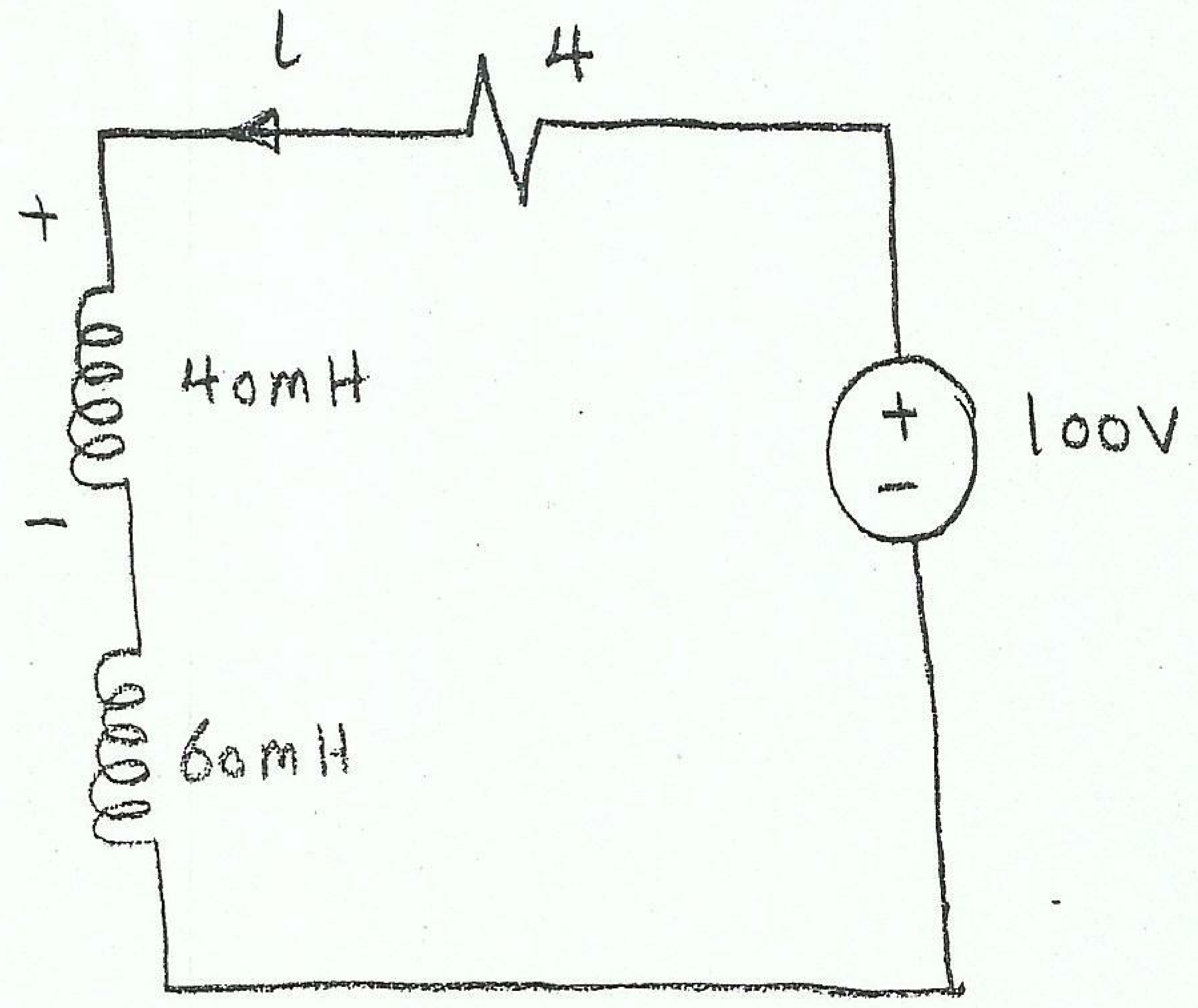
(10)

For $t<0$

$$\therefore i_{40mH}(0^-) = \frac{75}{5} = 15\,A = i_{40mH}(0^+)$$

For $t>0$

* using source transformation (2 times).



$$i_{40mH}(\infty) = \frac{100}{4} = 25A$$

$$\therefore \tau = \frac{L_{equv}}{R_{equv}} = \frac{100\,mH}{4} = 0.025\,s \quad , \frac{1}{\tau} = 40\,s^{-1}$$

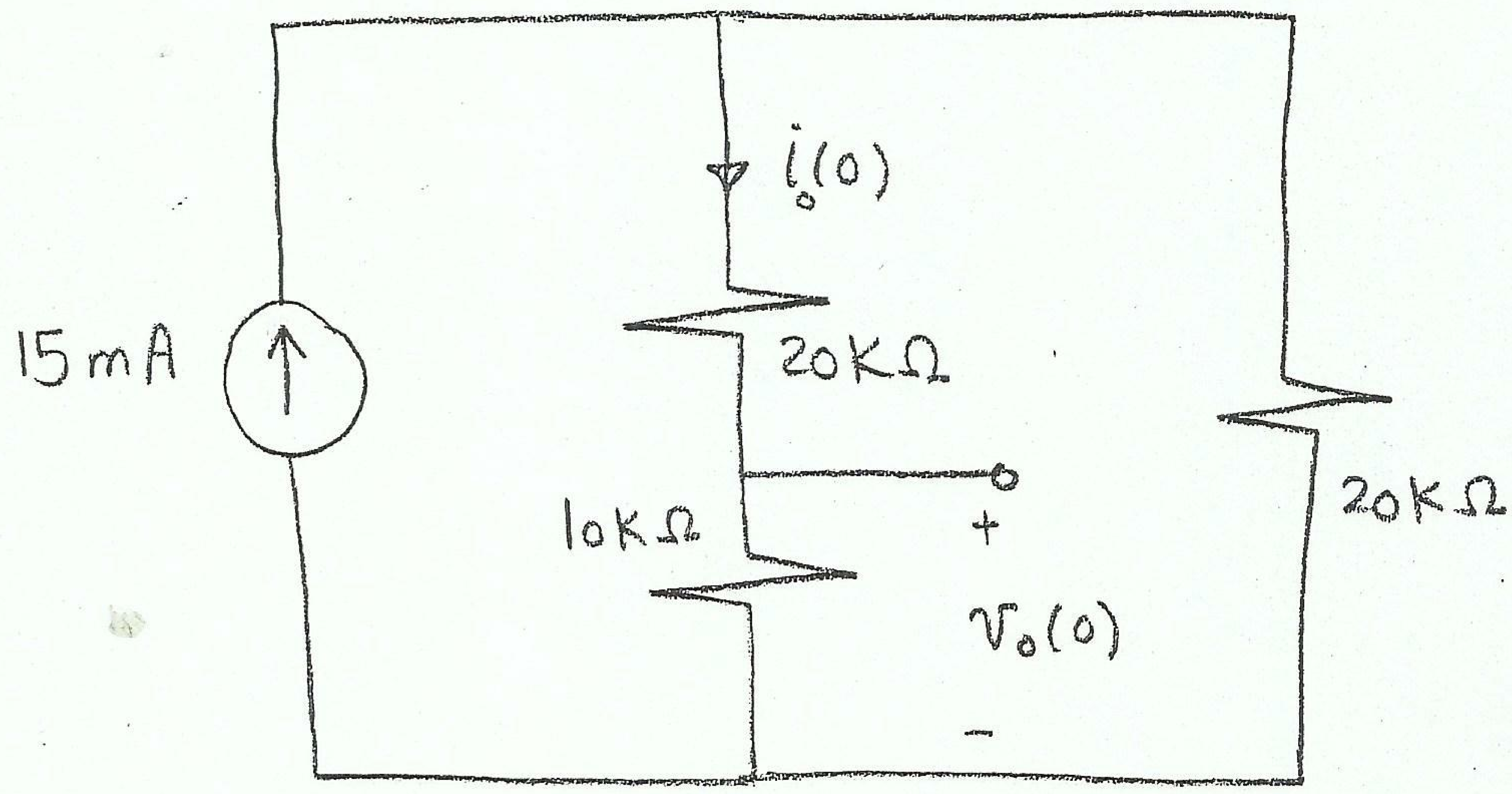
$$\therefore i(t) = 25 + (15-25)\,e^{-40t}$$

$$\therefore i(t) = 25 - 10\,e^{-40t} \quad , \quad t \geq 0$$

$$\boxed{\therefore v_o(t) = 0.04\,\frac{di(t)}{dt} = 16\,e^{-40t}\ V \quad , \quad t \geq 0}$$

(17)

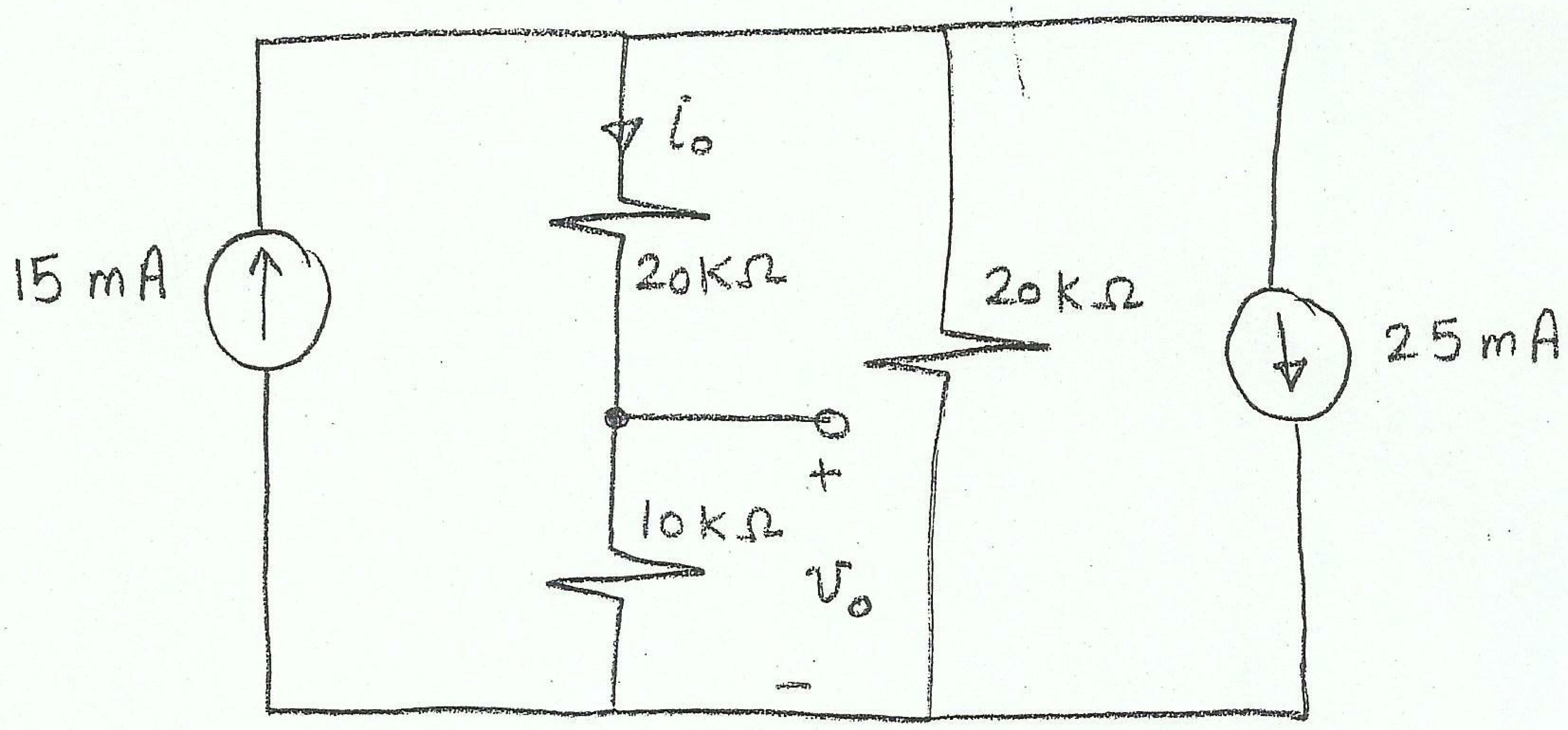
$t < 0$



$$i_o(\bar{0}) = 15mA * \frac{20}{20+30} = 6mA$$

$$\therefore v_o(\bar{0}) = 6 * 10 = 60V$$

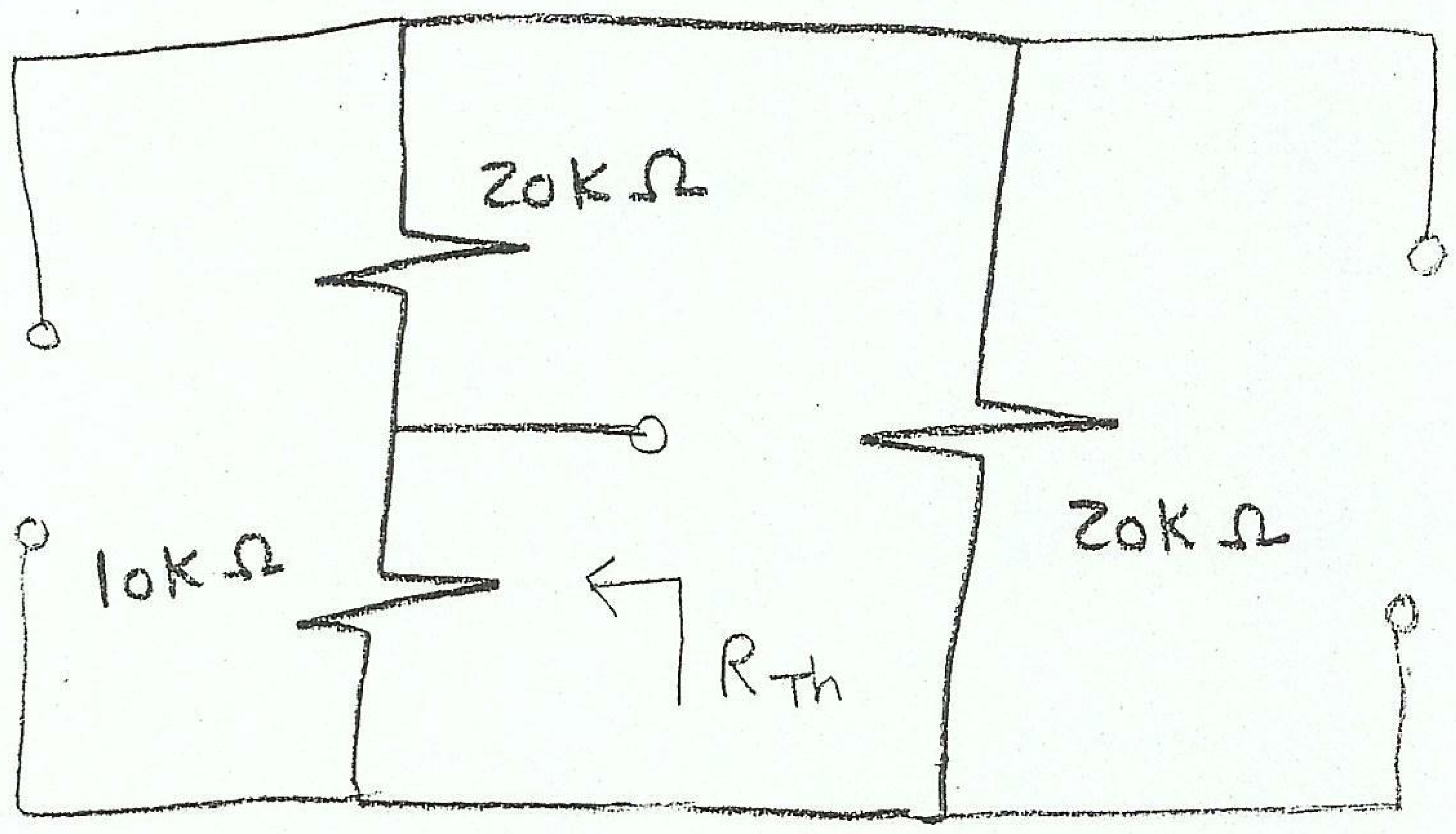
$0 \leq t < \infty$



$$\therefore i_o(\infty) = -10mA * \frac{20}{20+30} = -4mA$$

$$\therefore v_o(\infty) = -4 * 10 = -40V$$

$R_{Th} = 10^K \,//\, (20^K + 20^K)$

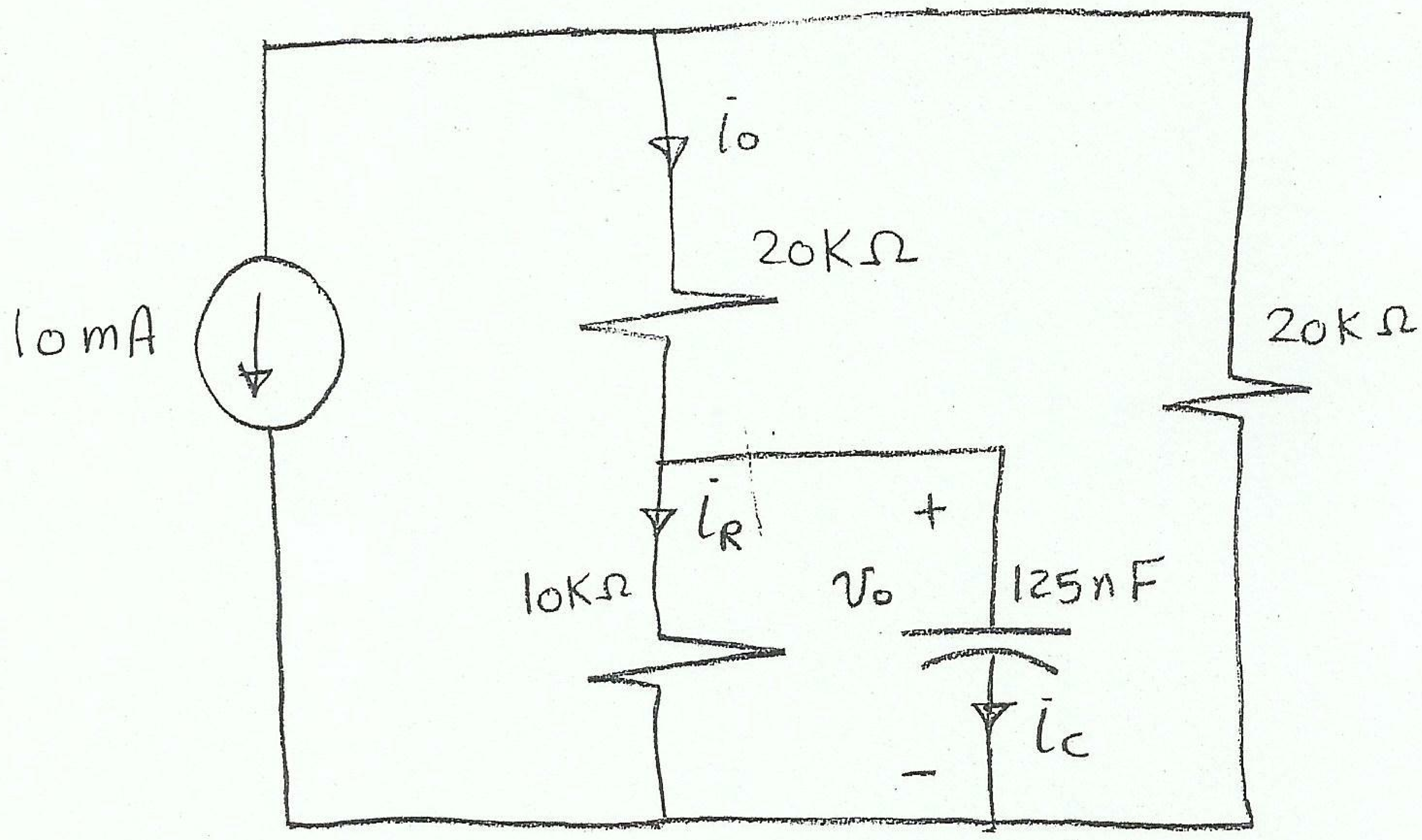


$\therefore R_{Th} = 8\,K\Omega$

$\therefore \tau = 8K * 125\,nF = 1\,ms$

$\therefore \frac{1}{\tau} = 1000\ s^{-1}$

$$\boxed{\therefore v_o(t) = -40 + 100\,e^{-1000t}\ V\ ,\ t \geq 0^+}$$



$\because i_c = C\frac{dV}{dt}$

$\therefore i_c = -12.5\,e^{-1000t}\ mA$

$\therefore i_R = \frac{v_o(t)}{10\,K\Omega} = -4 + 10\,e^{-1000t}\ mA$

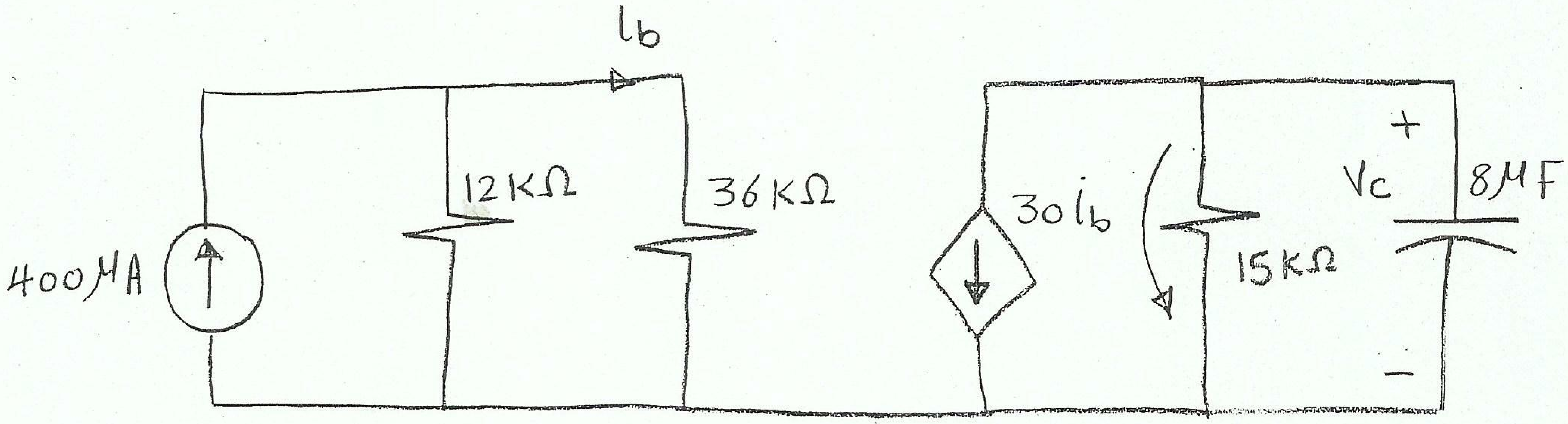
$$\boxed{\therefore i_o = i_c + i_R = -(4 + 2.5\,e^{-1000t})\ mA\ ,\ t \geq 0^+}$$

(12.)

For $t<0$

$i_b = 0$ (short cct) $\therefore V_c(0^-) = 0$

For $t>0$



$$\therefore i_b = 400\mu * \frac{12}{12+36} = 100 \mu A$$

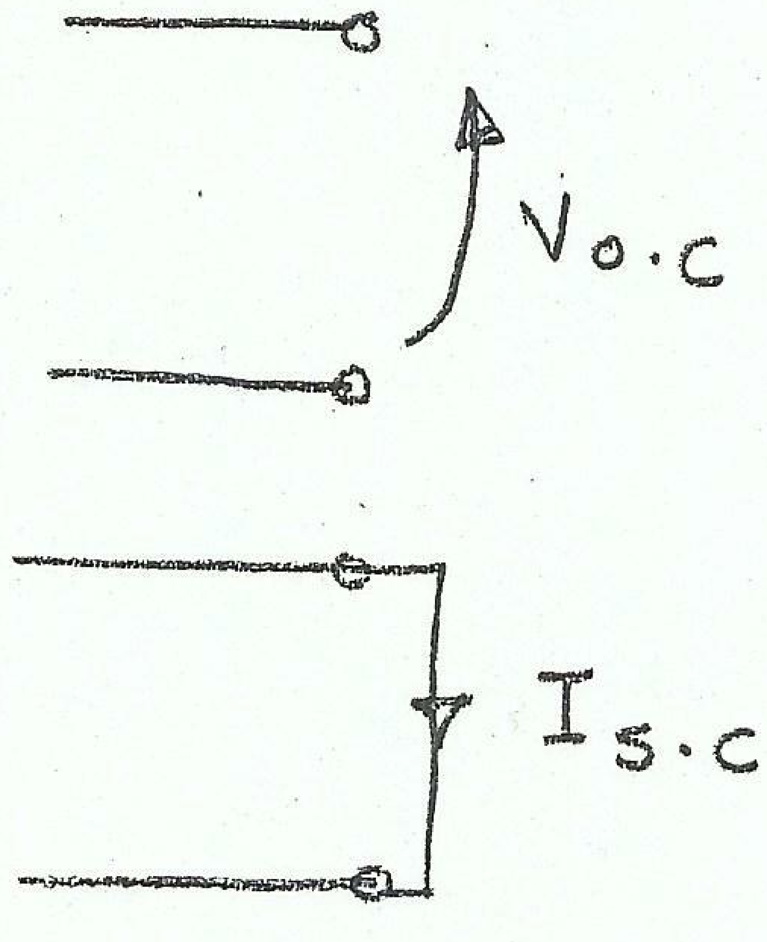
$$\therefore V_c(\infty) = -30 i_b * 15 K\Omega = -30 * 100 * 10^{-6} * 15 * 10^{3}$$

$$\therefore V_c(\infty) = -45 V$$

$$* \; R_{Th} = \frac{V_{Th}}{I_n} = \frac{V_{o.c}}{I_{s.c}}$$

$\because V_{o.c} = -45 V$, $I_{s.c} = -30 i_b = -3 mA$

$$\therefore R_{Th} = \frac{-45}{-0.003} = 15 K\Omega$$



$\therefore \tau = 15K * 8\mu F = 120 ms$

$$\therefore \frac{1}{\tau} = 8.33$$

$$\therefore v_c(t) = -45 + (0-(-45))\, e^{-8.33t}$$

$$\boxed{\therefore v_c(t) = -45\,(1 - e^{-8.33t}) \quad , t \geq 0}$$

$$\therefore W(t) = \frac{1}{2} C v^2(t) = \frac{1}{2} * 8*10^{-6} * v_c^2(t)$$

$$\therefore W(t) = 8100\,(1 - 2e^{-8.33t} + e^{-16.67t})\ \mu J$$

$$\therefore W(\infty) = 8100\ \mu J$$

$$\therefore \cancel{8100}(1 - 2e^{-8.33t} + e^{-16.67t}) = 0.9 * \cancel{8100}$$

let $x = e^{-8.33t}$

$$\therefore 1 - 2x + x^2 = 0.9$$

$$\therefore x = 1.9487 \ , \ 0.0513$$

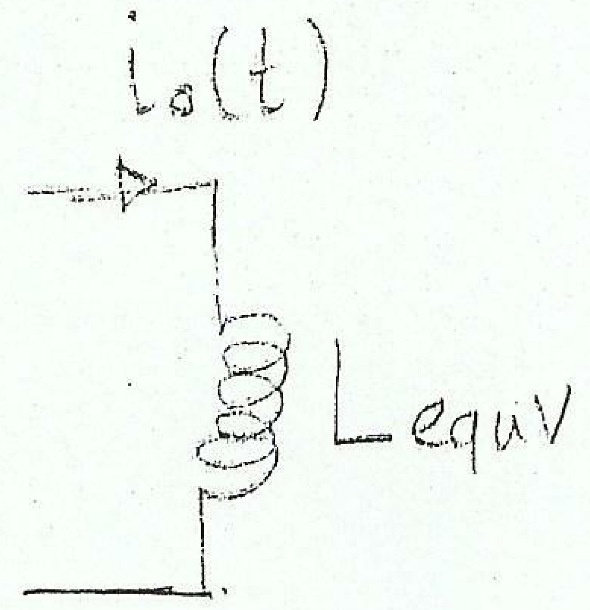
$$\therefore e^{-8.33t} = 0.0513 \qquad \boxed{\therefore t = 356.4\ ms}$$

(13)

(a) $L_{equv} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \dfrac{0.125 - 0.0625}{0.75 + 0.5} = 50\,mH$

$\tau = \dfrac{L}{R} = \dfrac{1}{5000}\,s \quad , \quad \dfrac{1}{\tau} = 5000\,s^{-1}$

$\because\ i_o(0) = 0 \quad , \quad i_o(\infty) = \dfrac{10}{250} = 40\,mA$



$$\boxed{\therefore\ i_o(t) = 40\,(1 - e^{-5000t})\ mA}$$

(b) $V_o = 10 - 250\, i_o = 10 - 250 * \dfrac{40}{1000}\,(1 - e^{-5000t})$

$$\boxed{\therefore\ V_o(t) = 10\, e^{-5000t}\ V}$$

or $V_o(t) = L_{equv}\, \dfrac{d\,i_o(t)}{dt}$

(c) $\because\ V_o = V_{L_1} = 0.5\,\dfrac{di_1}{dt} - 0.25\,\dfrac{di_2}{dt} = 10\,e^{-5000t}$

$\because\ i_o = i_1 + i_2$

$\therefore\ \dfrac{di_o}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt} = 200\,e^{-5000t}$

$\therefore\ \dfrac{di_2}{dt} = 200\,e^{-5000t} - \dfrac{di_1}{dt}$

$\therefore\ 10\,e^{-5000t} = 0.5\,\dfrac{di_1}{dt} - 50\,e^{-5000t} + 0.25\,\dfrac{di_1}{dt}$

$$\therefore 0.75 \frac{di_1}{dt} = 60\, e^{-5000t}$$

$$\therefore di_1 = 80\, e^{-5000t}\, dt$$

$$\therefore i_1 = 80 \int_0^t e^{-5000t}\, dt$$

$$\boxed{\therefore i_1(t) = 16\,(1 - e^{-5000t})\ \text{mA}, \quad t \geq 0}$$

$$(d) \therefore i_2 = i_o - i_1 = \left[40\,(1 - e^{-5000t})\right] - \left[16\,(1 - e^{-5000t})\right]$$

$$\boxed{\therefore i_2(t) = 24\,(1 - e^{-5000t})\ \text{mA}, \quad t \geq 0}$$

(14)

(a) $L_{equv} = 4 + 8 - 2 * 5 = 2\ H$

$$\tau = \frac{L}{R} = \frac{2}{50} = \frac{1}{25}\ s \quad \therefore \frac{1}{\tau} = 25\ s^{-1}$$

$$\because i(0) = 0 \quad , \because i(\infty) = \frac{200}{50} = 4A$$

$$\boxed{\therefore i(t) = 4(1 - e^{-25t})\ A}$$

(b) $$v_1(t) = 4\frac{di}{dt} - 5\frac{di}{dt} = -\frac{di}{dt}$$

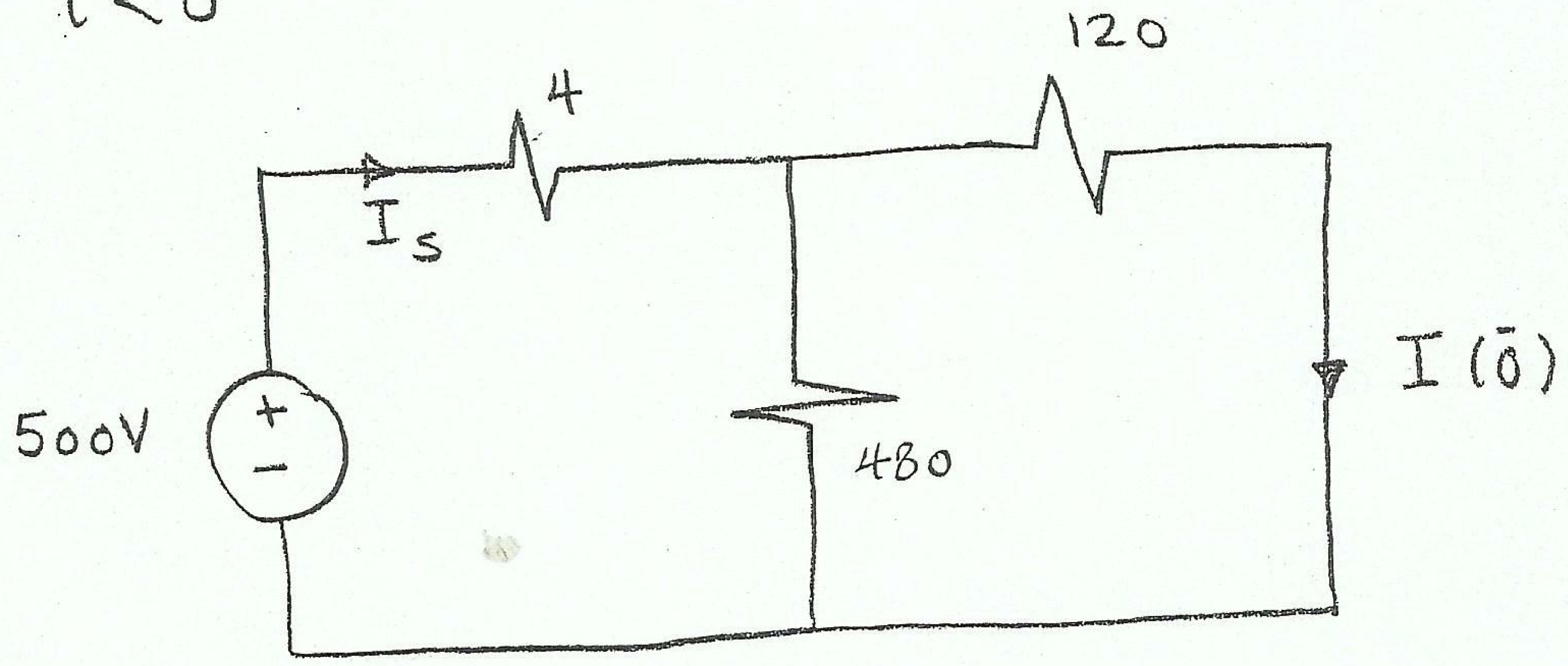
$$\boxed{\therefore v_1(t) = -100\, e^{-25t}\ V \quad , \quad t \geq 0^+}$$

(c) $$v_2(t) = 8\frac{di}{dt} - 5\frac{di}{dt} = 3\frac{di}{dt}$$

$$\boxed{\therefore v_2(t) = 300\, e^{-25t} \quad , \quad t \geq 0^+}$$

(15)

For $t<0$

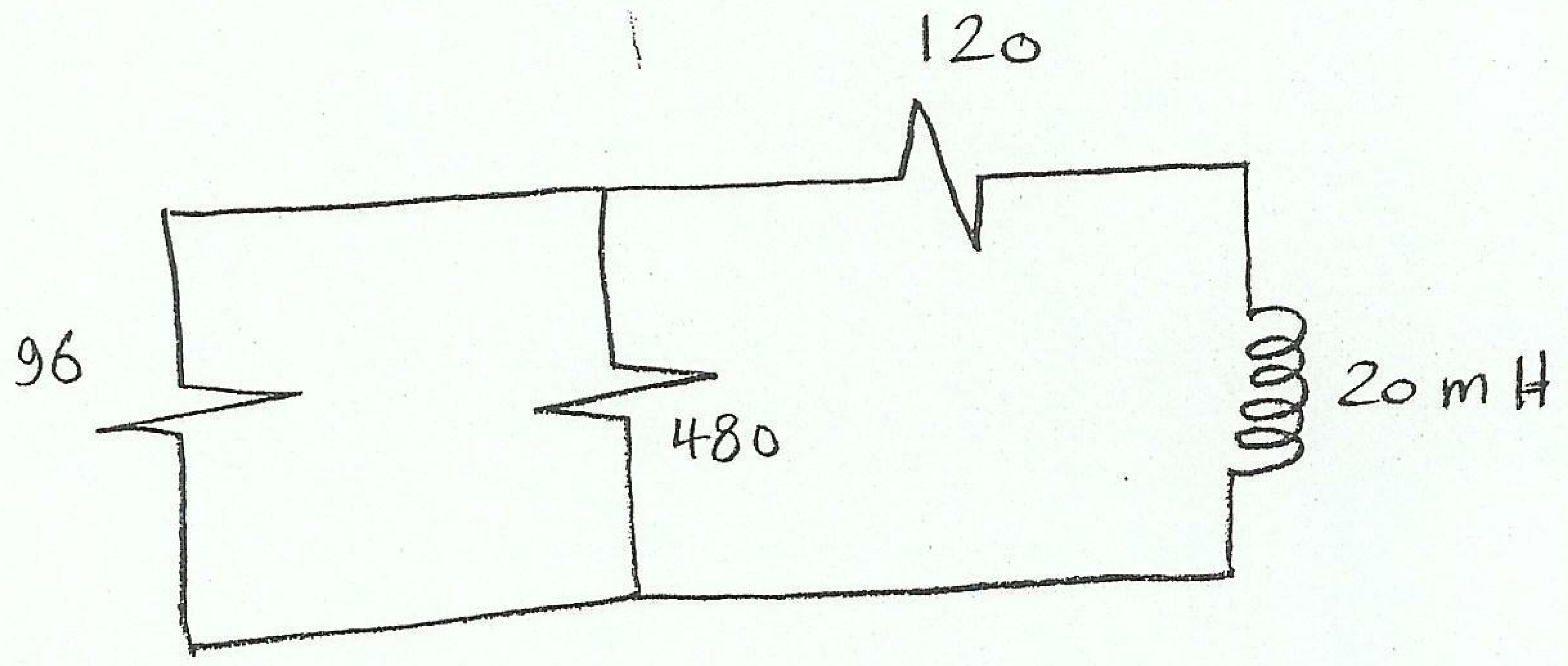


$$I_s = \frac{500}{4 + (120 // 480)} = 5A$$

$$\boxed{\therefore I(0^-) = 5 * \frac{480}{480+120} = 4A = I(0^+)}$$

For $0 \leq t \leq 100 \mu s$

$$i(t) = 4 e^{-t/\tau}$$



$R_{Th} = (96 // 480) + 120$

$\therefore R_{Th} = 200 \Omega$

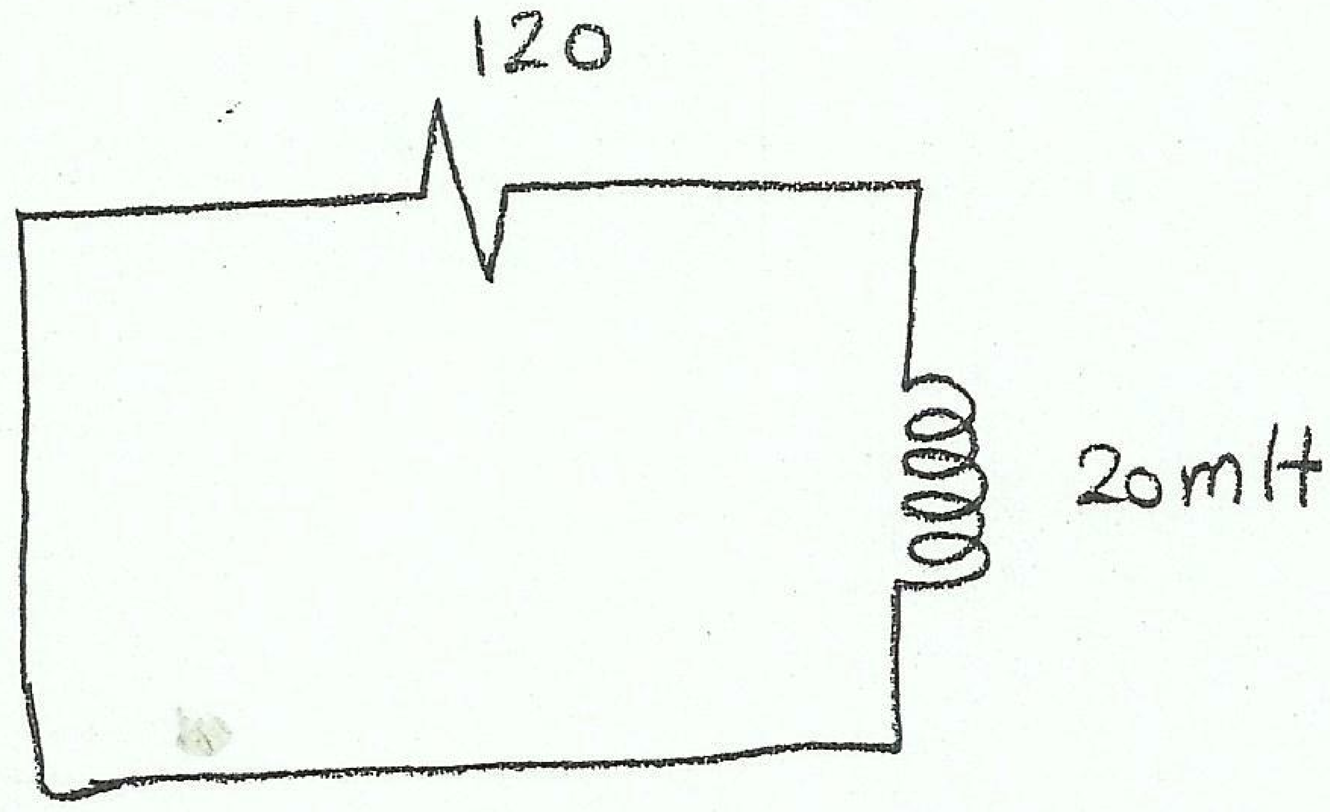
$$\therefore \frac{1}{\tau} = \frac{R}{L} = \frac{200}{0.02} = 10,000 \ s^{-1}$$

$$\therefore i(t) = 4 e^{-10,000 t}$$

$$\boxed{\therefore i(25 \mu s) = 4 e^{-10^4 * 25 * 10^{-6}} = 3.12 A}$$

$\therefore\ i(100\mu s) = 4e^{-10^4 * 10^{-4}} = 4e^{-1} = 1.47\ A$

For $100\mu \leqslant t < \infty$



$$\frac{1}{\tau} = \frac{R}{L} = \frac{120}{0.02} = 6000\ s^{-1}$$

$$\therefore\ i(t) = 1.47\ e^{-6000t}\ A$$

$$\therefore\ i(200\mu s) = 1.47\ e^{-6*10^3 * 200*10^{-6}} = 1.47\ e^{-0.6}$$

$$\boxed{\therefore\ i(200\mu s) = 807.59\ mA}$$

When $0 \leqslant t \leqslant 100\mu s$

$$\therefore\ i(t) = 4\ e^{-10{,}000t}$$

$$\therefore\ v_L(t) = L\frac{di}{dt} = 0.02 * 4 * -10^4\ e^{-10^4 t}$$

$$\therefore\ v_L(t) = -800\ e^{-10^4 t}\ V$$

$$\therefore\ v_L(100^- \mu s) = -800 * e^{-10^4 * 10^{-4}} = -800\ e^{-1}$$

$$\boxed{\therefore\ v_L(100^- \mu s) = -294.3\ V}$$

When $100\mu s \ll t < \infty$

$$\therefore i(t) = 1.47\, e^{-6000(t - 100*10^{-6})}$$

$$\therefore V_L = 0.02 * 1.47 * (-6000)\, e^{-6000(t - 100*10^{-6})}$$

$$\therefore V_L(t) = -176.58\, e^{-6000(t - 10^{-4})} \ V$$

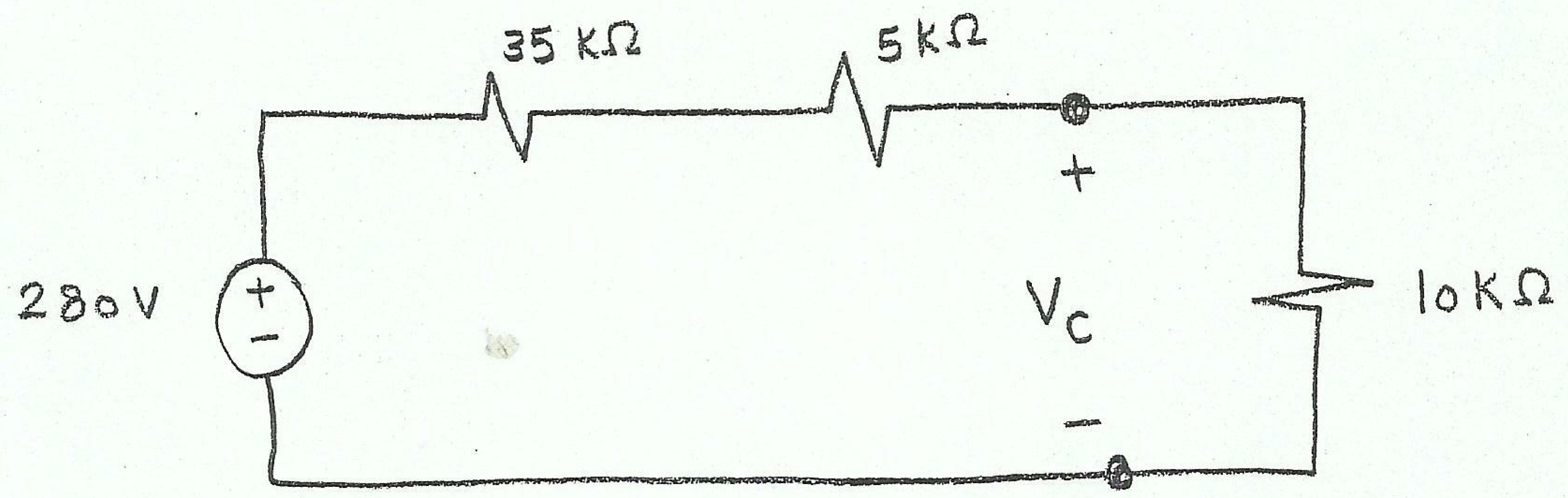
$$\therefore V_L(100^{+}\mu s) = -176.58\, e^{-600(10^{-4} - 10^{-4})}$$

$$\boxed{\therefore V_L(100^{+}\mu s) = -176.58V}$$

(16)

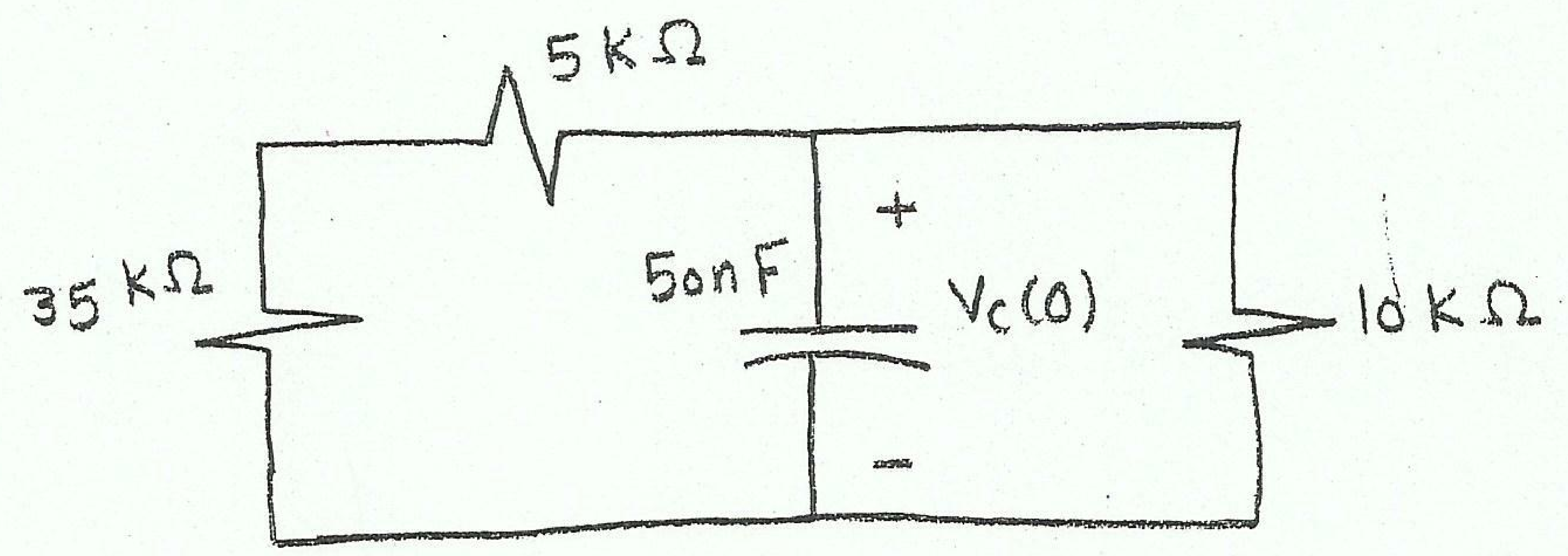
For $t<0$

* using Source Transformation:



$$\therefore V_c(0) = 280 * \frac{10}{10+40} = 56\ V$$

For $0 \leq t \leq 400\ \mu s$



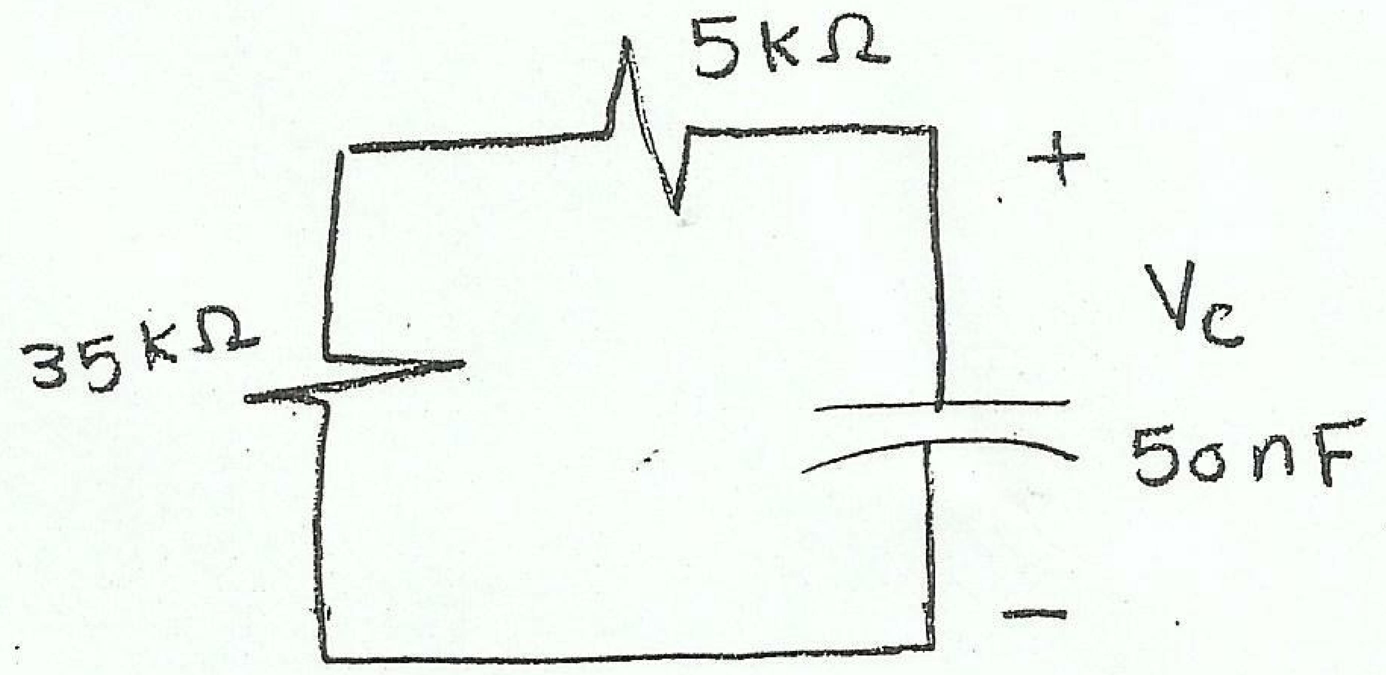
$$\tau = R_{Th} * C \quad , \quad R_{Th} = (5+35) // 10 = 8\,K\Omega$$

$$\therefore \tau = 8*10^{3} * 50*10^{-9} = 400\ \mu s$$

$$\therefore V_c(t) = 56\ e^{-t/\tau}$$

$$\therefore V_c(400\mu s) = 56 * e^{-1} = 20.6\ V$$

For $400\mu < t < 1.4\ ms$

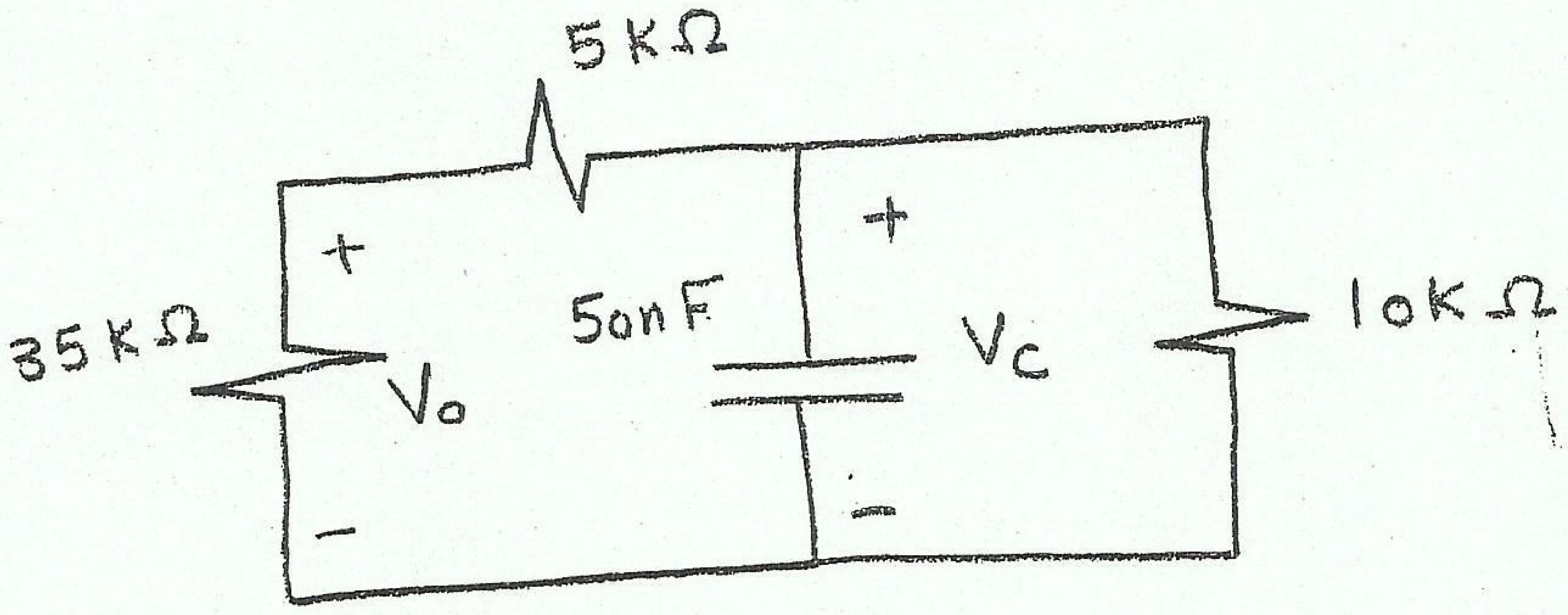


$$\tau = (40\,K\Omega)\cdot(50\,nF) = 2\,ms \quad , \quad \frac{1}{\tau} = 500\ s^{-1}$$

$$\therefore V_c(t) = 20.6\ e^{-500\,(t - 400 \times 10^{-6})}\ V$$

$$\boxed{\therefore V_c(1.4\,ms) = 20.6\ e^{-500\,(1400 - 400) \times 10^{-6}} = 12.5\ V}$$

For $1.4\,ms < t < \infty$



$$\tau = 400\,\mu s \, , \quad \frac{1}{\tau} = 2500\ s^{-1}$$

$$\therefore V_c(t) = 12.5\ e^{-2500\,(t - 1.4 \times 10^{-3})}\ V$$

$$\therefore V_c(1.6\,ms) = 12.5\ e^{-2500\,(1.6 - 1.4) \times 10^{-3}} = 7.58\,V$$

$$\boxed{\therefore V_o(1.6\,ms) = 7.58 * \frac{35}{35+5} = 6.63\,V}$$