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A comprehensive design and performance evaluation study of counter flow wet cooling towers

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Abstract

Fouling of cooling tower fills is one of the most important factors affecting its thermal performance, which reduces cooling tower effectiveness and capability with time. In this paper, the fouling model presented in an earlier paper using the experimental data on fill fouling, is used to investigate the risk based thermal performance of the cooling tower. It is demonstrated that effectiveness of the cooling tower degrades significantly with time indicating that for a low risk level ($p = 0.01$), there is about 6.0% decrease in effectiveness for the given fouling model. The sensitivity analysis of the cooling tower is investigated for both rating and design calculation for different values of mass flow rate ratios. The effect of atmospheric pressure on the thermal performance of the cooling tower is also demonstrated.

Keywords: Design; Cooling tower; Calculation; Performance; Fouling

Etude permettant d'évaluer la conception et al performance des tours de refroidissement humides à contre courant

Mots-clés: Conception; Tour de refroidissement; Calcul; Performance; Encrassement

1. Introduction

Cooling towers are designed to cool a warm water stream through evaporation of some of the water into an air stream. The towers are commonly used in large thermal systems to reject the waste heat from the systems via a water loop between the two devices. One of the advantages of the cooling tower over the dry heat exchanger is that, through evaporation, the circulating water temperature may approach the atmospheric wet bulb temperature rather than the dry bulb temperature. There are several types of cooling towers. Probably the most common is the mechanical draft tower in which the water enters at the top of the tower as a spray and flows downward through the tower. Ambient air is drawn into the tower with the help of fans, and flows in a counter or cross-current manner to the water stream. If the fans are at the bottom of the tower and blow the air upward past the water flow, the tower is termed as a forced draft tower, whereas if the fans are at the top, it is an induced draft tower. Large-size atmospheric towers do not use a fan and rely on the buoyancy effect of the heated

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Nomenclature

A_V	surface area of water droplets per unit volume of the tower, $m^2\ m^{-3}$
c_{pa}	specific heat at constant pressure of moist air, $kJ\ kg_a^{-1}\ K^{-1}$
C_1	constant used in Eq. (2)
C_2	constant used in Eq. (2), $kg\ m^{-3}$
h	specific enthalpy of moist air, $kJ\ kg_a^{-1}$
h_c	convective heat-transfer coefficient of air, $kW\ m^{-2}\ K^{-1}$
h_D	convective mass-transfer coefficient, $kg_w\ m^{-2}\ s^{-1}$
h_f	specific enthalpy of saturated liquid water, $kJ\ kg_w^{-1}$
$h_{f,w}$	specific enthalpy of water evaluated at t_w, $kJ\ kg_w^{-1}$
h_g	specific enthalpy of saturated water vapor, $kJ\ kg_w^{-1}$
h_g^0	specific enthalpy of saturated water vapor evaluated at 0 °C, $kJ\ kg_w^{-1}$
$h_{fg,w}$	change-of-phase enthalpy of water evaluated at t_w ($h_{fg,w} = h_{g,w} - h_{f,w}$), $kJ\ kg_w^{-1}$
Le	Lewis number ($Le = h_c/h_D c_{pa}$)
$\dot{m}_a$	mass flow rate of air, $kg_a\ s^{-1}$
$\dot{m}_w$	mass flow rate of water, $kg_w\ s^{-1}$
m_{ratio}	$\dot{m}_w/\dot{m}_a$
M	median weight to reach critical level of fouling, $kg\ m^{-3}$
NTU	number of transfer units
p	risk level
t	dry-bulb temperature of moist air, °C; time, s
t_w	water temperature, °C
U_Y	uncertainty in parameter Y, units of Y
U_{X_i}	uncertainty in parameter X_i units of X_i
V	volume of tower, m^3
w	weight density of fouling material, $kg\ m^{-3}$
W	humidity ratio of moist air, $kg_w\ kg_a^{-1}$
$\bar{X}$	nominal value of X, units of X
X_i	general input variable
Y	response parameter
$\bar{Y}$	nominal value of Y, units of Y
η_F	fill performance index, ($\eta_F = h_D A_V V/\dot{m}_w$)
$\alpha^{1/2}$	scatter in time
Φ	cumulative normal distribution function
ϕ	rate of deposition or removal, $m^2\ K\ J^{-1}$
ε	effectiveness

Subscripts

a	moist air
c	clean conditions
cal	calculated
cr	critical fouled conditions
d	deposition
db	dry-bulb
em	empirical
f	fouled conditions
F	fill
g,w	vapor at water temperature
i	inlet
norm	normalized
o	outlet
r	removal
s,w	saturated moist air at water temperature
w	water
wb	wet-bulb

air, and a nozzle-like (or hyperbolic) shape to cause air circulation. The cooling towers at conventional or nuclear power plants are of this type.

As with other heat exchangers, there are a variety of different flow arrangements. In a counter-flow tower the air flows upward as the water is flowing downward, while in a cross-flow tower the airflow is horizontal and perpendicular to the downward water flow. The inside of the tower is packed with fill or packing material having a large surface area, as shown in Fig. 1. A common fill material is wooden or fiberglass slats over which the water slowly drips. The purpose of the fill is to distribute the water flow and provide a large surface area for contact between the air and water; however, the fills are subjected to fouling during operation, which reduces tower effectiveness with time. It is important to note that Mohiuddin and Kant [1,2] described a detailed procedure for the thermal design of wet, counter-flow and cross-flow mechanical and natural draught cooling towers; however, they did not discuss the impact of fouling on their performance.

The objective of this paper is to present a risk based (or probabilistic) approach to the analysis of fouling models and to describe its impact on the thermal performance of the cooling towers. A cooling tower model in conjunction with the fouling model is used to study the effect of fouling on tower effectiveness and water outlet temperatures for a small size cooling tower operating under similar conditions.

2. Fouling growth model

The most widely accepted fouling model, which is used in conventional heat exchangers, is based on the following general material balance equation first proposed by Kern and Seaton [3]

$$\frac{dR_f(t)}{dt} = \phi_d - \phi_r \tag{1}$$

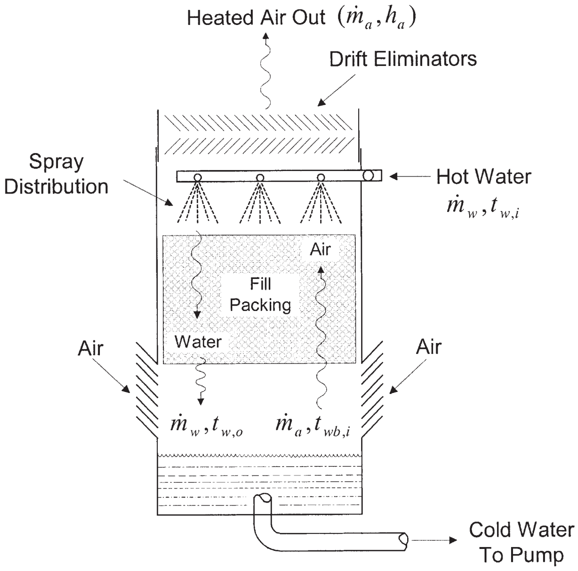

Fig. 1. Schematic of a counter-flow wet cooling-tower.

In the above equation, the rate of fouling deposition, ϕ_d, depends on the type of fouling mechanism (sedimentation, crystallization, organic material growth, etc.), while the rate of fouling removal, ϕ_r, depends on both the hardness and adhesive strength of the deposit and the shear stress due to the flow velocity, as well as the fill configuration. The rates of deposition and removal have been given many different forms in the heat exchanger literature (depending upon the type of fouling mechanism) by various investigators.

Fouling, as defined for cooling towers, is the process of deposition of foreign matter, including bio-growth, on the water film flow area. It inhibits the cooling process and allows excessive weight to build up in the cooling tower. In more severe circumstances; however, fouling can result in a reduction in the overall effectiveness of the tower—a symptom of fill fouling interfering with air and water flow through the tower. It is important to note that plastic fills are more prone to fouling than traditional splash bars.

In monitoring fouling in cooling towers, the weight gain with respect to time is easy to conduct on any cooling tower site. Weight gives a good indication of the fill pack tendencies with regard to fouling, but does not directly address the cooling tower users' main interest; that is, performance degradation due to fill fouling. The users of cooling towers are mainly interested in knowing; (i) the correlation between the weight gain and fouling, (ii) loss in thermal performance, (iii) the effect of weight gain on fill pressure drop, and fill performance index ($\eta_F = h_D A_V V/\dot{m}_w$). Khan and Zubair [4], using the experimental data reported in Michel et al. [5] have recently developed a model showing a correlation between normalized fill performance index due to fouling ($\eta_{F,norm}$) and weight

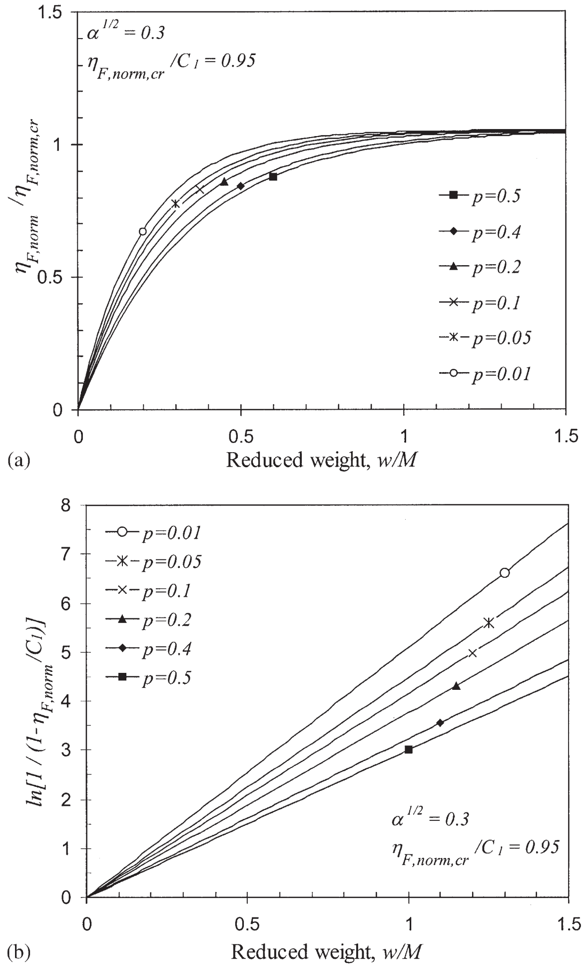

Fig. 2. Normalized fill performance index versus reduced weight (w/M); (a) non-transformed coordinate system (b) transformed coordinate system.

gain (w). The model is of the form [4]

$$\eta_{F,norm} = \left(\frac{h_D A_V V}{\dot{m}_w}\right)_{norm} = \frac{\left(\left(\frac{h_D A_V V}{\dot{m}_w}\right)_c - \left(\frac{h_D A_V V}{\dot{m}_w}\right)_f\right)}{\left(\frac{h_D A_V V}{\dot{m}_w}\right)_c} = C_1(1 - \exp(-w/C_2)) \quad (2)$$

where C_1 and C_2 are constants depending on the fouling characteristics of the tower. C_1 represents the increase in value of $\eta_{F,norm}$ when the fouling reaches its asymptotic value, and C_2 represents the weight gain constant indicating that the fill performance index has decreased to 63.2% of the asymptotic value of weight gain due to fouling. A linear

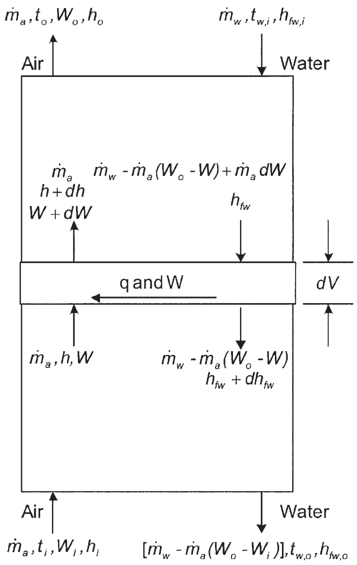

Fig. 3. Mass and energy balance of a counter-flow wet cooling-tower.

description of the above model can be expressed as

$$\ln\left(\frac{1}{1 - \dfrac{\eta_{\mathrm{F,norm}}}{C_1}}\right) = w/C_2 \tag{3}$$

The constant C_2 is expressed in terms of the critical acceptable value of the fill performance index $\eta_{\mathrm{F,norm,cr}}$ as

$$C_2 = w_{\mathrm{cr}}/\ln[1/(1 - \eta_{\mathrm{F,norm,cr}}/C_1)] \tag{4}$$

where the critical value of weight gain due to fouling w_{cr}, is expressed as

$$w_{\mathrm{cr}} = M/[1 - \sqrt{\alpha}\Phi^{-1}(p)] \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (3) and rearranging, we get

$$\begin{aligned} &\eta_{\mathrm{F,norm}}(w, p; \sqrt{\alpha}) \\ &\quad = C_1[1 - \exp\{-\ln[1/(1 - \eta_{\mathrm{F,norm,cr}}/C_1)][1 \\ &\qquad - \sqrt{\alpha}\Phi^{-1}(p)](w/M)\}] \end{aligned} \tag{6}$$

Note that the parameters M and $\alpha^{1/2}$ represent the median weight and the scatter parameter in a transformed coordinate system, where the exponential fouling growth model may be treated as a linear replica.

Fig. 2(a) and (b) shows the plot of normalized fill performance index as a function of reduced weight (w/M) and the risk level (p), representing the probability of fill surface being fouled up to a critical level after which a cleaning is needed; and for the scatter parameter $\alpha^{1/2} = 0.3$. In plotting this figure, we have considered the critical normalized fill performance index as 95% that of the asymptotic value. This critical value may be considered as a representative number for most cooling towers when it becomes necessary to clean the tower. As expected, this figure also shows that, for a low risk level, the fouling weight required to reach the critical level is faster compared with the deterministic case (i.e., $p = 0.50$). We notice that the scatter in the growth rate is much different in the transformed coordinate system (Fig. 2(b)) when compared with the non-transformed coordinate system (Fig. 2(a)).

3. Cooling tower model

A schematic of a counter-flow cooling-tower showing the important states is presented in Fig. 3. The major assumptions that are used to derive the basic modeling equations may be summarized as [6,7]:

- heat and mass-transfer in a direction normal to the flows only;
- negligible heat and mass transfer through the tower walls to the environment;
- negligible heat transfer from the tower fans to air or water streams;
- constant water and dry air specific heats;
- constant heat and mass transfer coefficients throughout the tower;
- constant value of Lewis number throughout the tower;
- water lost by drift is negligible;
- uniform temperature throughout the water stream at any cross section; and
- uniform cross-sectional area of the tower.

From steady-state energy and mass balances on an incremental volume (Fig. 3), we get [7]

$$\dot{m}_{\mathrm{a}}\mathrm{d}h = -[\dot{m}_{\mathrm{w}} - \dot{m}_{\mathrm{a}}(W_{\mathrm{o}} - W)]\mathrm{d}h_{\mathrm{f,w}} + \dot{m}_{\mathrm{a}}\mathrm{d}Wh_{\mathrm{f,w}} \tag{7}$$

We may also write the water energy balance in terms of the heat- and mass-transfer coefficients, h_{c} and h_{D}, respectively, as

$$-\dot{m}_{\mathrm{w}}\mathrm{d}h_{\mathrm{f,w}} = h_{\mathrm{c}}A_{\mathrm{V}}\mathrm{d}V(t_{\mathrm{w}} - t) + h_{\mathrm{D}}A_{\mathrm{V}}\mathrm{d}V(W_{\mathrm{s,w}} - W)h_{\mathrm{fg,w}} \tag{8}$$

and the air side water-vapor mass balance as

$$\dot{m}_{\mathrm{a}}\mathrm{d}W = h_{\mathrm{D}}A_{\mathrm{V}}\mathrm{d}V(W_{\mathrm{s,w}} - W) \tag{9}$$

By substitution Lewis number as $\mathrm{Le} = h_{\mathrm{c}}/h_{\mathrm{D}}c_{\mathrm{pa}}$ in Eq. (8), we get after simplification

$$-\dot{m}_{\mathrm{w}}\mathrm{d}h_{\mathrm{f,w}} = h_{\mathrm{D}}A_{\mathrm{V}}\mathrm{d}V[\mathrm{Le}\ c_{\mathrm{pa}}(t_{\mathrm{w}} - t) + (W_{\mathrm{s,w}} - W)h_{\mathrm{fg,w}}] \tag{10}$$

It should be noted that we have defined Lewis number in Eq. (10), similar to the definition that is used by Braun et al. [8] and Kuehn et al. [6]; however Jaber and Webb [9] and El-Dessouky et al. [10] have used $\mathrm{Le} = Sc/\mathrm{Pr}$, commonly used in heat- and mass-transfer literature. In this regard, we prefer

Table 1
Experimental and calculated values of cooling tower parameters

$t_{w,i}$ (°C)	$t_{w,o}$ (°C)	$t_{db,i}$ (°C)	$t_{wb,i}$ (°C)	$\dot{m}_a$ (kg/s)	$\dot{m}_w$ (kg/s)	NTU_{em}	NTU_{cal}	$t_{wb,o}^{clac}$ (°C)	$t_{wb,o}^{exp}$ (°C)
31.22	23.88	37.05	21.11	1.158	0.754	1.297	1.254	26.35	26.05
41.44	26.00	34.11	21.11	1.158	0.754	1.297	1.226	31.07	31.16
28.72	24.22	29.00	21.11	1.187	1.259	1.745	1.794	26.32	26.17
34.50	26.22	30.50	21.11	1.187	1.259	1.745	1.726	29.98	29.90
38.78	29.33	35.00	26.67	1.265	1.008	1.467	1.484	33.02	32.89
38.78	29.33	35.00	26.67	1.250	1.008	1.477	1.512	33.09	32.89

to stick to the notation of Kuehn et al. [6] that is considered as one of the standard references in the cooling tower literature. Combining Eqs. (7)–(9) and (10), we get after some simplification [7]

$$\frac{dh}{dW} = \mathrm{Le}\frac{(h_{s,w} - h)}{(W_{s,w} - W)} + (h_{g,w} - h_g^0\mathrm{Le}) \tag{11}$$

It should be noted that Eq. (11) describes the condition line on the psychrometric chart for the changes in state for moist air passing through the tower. For given water temperatures ($t_{w,i}, t_{w,o}$), Lewis number (Le), inlet condition of air and mass flow rates, Eqs. (7) and (11) may be solved numerically for exit conditions of both the air and water stream. The solution is iterative with respect to the air humidity ratio and temperatures (W, t and t_w). Eqs. (7), (9) and (11) can be integrated numerically, over the entire tower volume from air inlet to outlet by a procedure similar to that described in Kuehn et al. [6] and Khan and Zubair [7].

A computer program is written in Engineering Equation Solver (EES) for solving Eqs. (7), (9) and (11). In this program, properties of air–water vapor mixture are needed at each step of the numerical calculation. These properties are obtained from the built-in functions provided in EES. The program gives the dry-bulb temperature and wet-bulb temperature of air, as well as water temperature and humidity ratio of air at each step of numerical calculation starting from air-inlet to air-outlet values. If the value of ($h_D A_V$) is known, the required tower volume may be obtained by using [6,7]

$$V = \frac{\dot{m}_a}{h_D A_V}\int_{W_i}^{W_o}\frac{dW}{W_{s,w} - W} \tag{12}$$

The integral in the above equation is solved numerically. The number of transfer units of the tower is calculated by [7]

$$NTU_{cal} = \frac{h_D A_V V}{\dot{m}_a} = \int_{W_i}^{W_o}\frac{dW}{W_{s,w} - W} \tag{13}$$

The cooling tower effectiveness (ε), which is defined as the ratio of actual energy to the maximum possible energy transfer, is given by

$$\varepsilon = \frac{h_o - h_i}{h_{s,w,i} - h_i} \tag{14}$$

The correlations for heat and mass transfer of cooling towers in terms of physical parameters are not easily available. It is typical to correlate the tower performance data for specific tower designs. For instance, mass transfer data are typically correlated in the form [11]

$$\frac{h_D A_V V}{\dot{m}_w} = c\left(\frac{\dot{m}_w}{\dot{m}_a}\right)^n \tag{15}$$

where c and n are empirical constants specific to a particular tower design. Multiplying both sides of the above equation by ($\dot{m}_w/\dot{m}_a$) and considering the definition for NTU (refer to Eq. (13)) gives the empirical value of NTU as

$$NTU_{em} = c\left(\frac{\dot{m}_w}{\dot{m}_a}\right)^{n+1} \tag{16}$$

The coefficients c and n of the above equation were fit to the measurements of Simpson and Sherwood [12] for four different tower designs over a range of performance conditions by Braun et al. [8]. Their experimental values were also compared with the values obtained by our model, and the results are discussed in Table 1. It was shown that the calculated and empirical values of NTU are well within the acceptable limits. Also the wet-bulb temperature of outlet air ($t_{wb,o}$), calculated from the present model is compared with the experimental values reported in Simpson and Sherwood [12], the two values are very close to each other (within $\pm$0.26%).

4. Effect of fouling on thermal performance

The cooling tower model discussed in Section 3 can be used for design and rating calculations of a counter flow wet cooling tower. It should be noted that the rating calculations procedure is used for studying the effect of fouling on cooling tower performance. In rating calculations, water outlet temperature ($t_{w,o}$) and tower effectiveness (ε) are

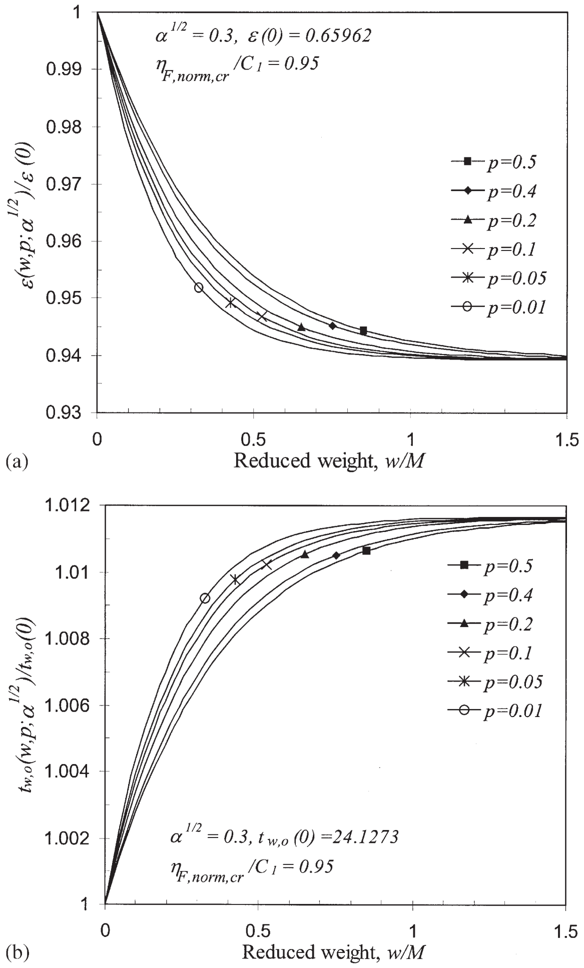

Fig. 4. Normalized tower effectiveness (a) and reduced water outlet temperature (b) versus reduced weight (*w/M*).

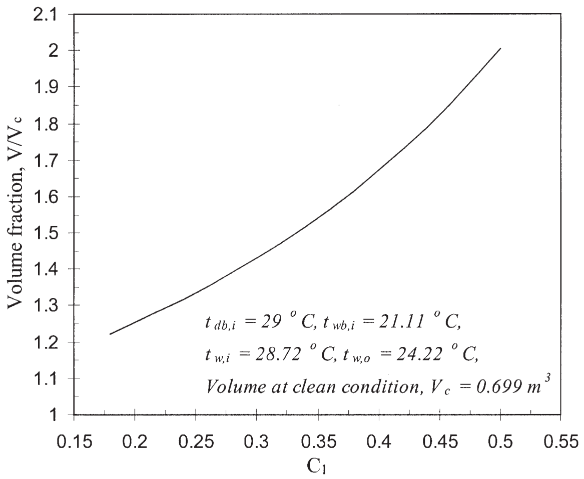

Fig. 5. Volume fractions versus the fouling constant C_1.

calculated for the following set of input conditions: inlet air temperatures [dry bulb ($t_{db,i}$) and wet bulb ($t_{wb,i}$)], water inlet temperature ($t_{w,i}$), mass flow rates [air ($\dot{m}_a$) and water ($\dot{m}_w$)], fill performance index (η_F) and tower volume (V). While in design calculations, the volume of the cooling tower is calculated for the following set of input conditions: inlet air temperatures [dry bulb ($t_{db,i}$) and wet bulb ($t_{wb,i}$)], water inlet temperature ($t_{w,i}$), mass flow rates [air ($\dot{m}_a$) and water ($\dot{m}_w$)], fill performance index (η_F), and water outlet temperature ($t_{w,o}$). The cooling tower model in combination with the fouling model is used for studying the thermal performance of the tower under fouled conditions. The time and risk dependent tower effectiveness of the cooling tower discussed in Section 3 are presented in Fig. 4(a) in a reduced coordinate system. The graph is drawn for the following set of input data given in Simpson and Sherwood [12]: $t_{db,i} = 29.0$ °C, $t_{wb,i} = 21.1$ °C, $t_{w,i} = 28.7$ °C, $V = 0.697$ m^3, Le $= 0.9$, $\dot{m}_w = 1.26$ kg/s and $\dot{m}_a = 1.19$ kg/s. In these figures, reduced tower effectiveness $\varepsilon(w,p;\sqrt{\alpha})/\varepsilon(0)$ versus reduced fouling weight w/M plots for different risk level p and scatter parameter $\alpha^{1/2} = 0.3$, are plotted for the fouling-growth model that we discussed earlier.

As expected, the effectiveness of the cooling tower degrades significantly with time indicating that for a low risk level ($p = 0.01$), there is about 6.0% decrease in effectiveness for the given fouling model. The variations in the reduced water outlet temperature versus reduced fouling weight for different risk levels p and for scatter parameter $\alpha^{1/2} = 0.3$, is shown in Fig. 4(b). As one would expect, the figures show that for a low risk level (i.e., high reliability) when compared with the deterministic case, the water outlet temperature is high, indicating that there will be a lower heat transfer rate due to fouling compared with the deterministic case. It is noticed that there is about 1.2% increase in water outlet temperature for the given fouling model.

We may notice from the above discussion that fouling reduces the performance of a cooling tower, which is reflected in the decreased value of the tower effectiveness. In order to achieve a constant value of the cooling tower effectiveness under fouled conditions its volume has to be increased, which is shown in Fig. 5. In this figure, a plot of the volume fraction (V_f/V_c) of the cooling tower is shown as a function of the constant C_1 that is introduced earlier in Eq. (2). It should be noted that the constant C_1 represents an increase in value of $\eta_{F,norm}$ when the fouling reaches its asymptotic value.

5. Effect of pressure

Sutherland [13] mentioned that an increase in altitude of approximately 850 m would result in, 10 kPa decrease of atmospheric pressure. This change in atmospheric pressure would definitely effect the operations of cooling tower because it directly influences the wet-bulb temperature. It

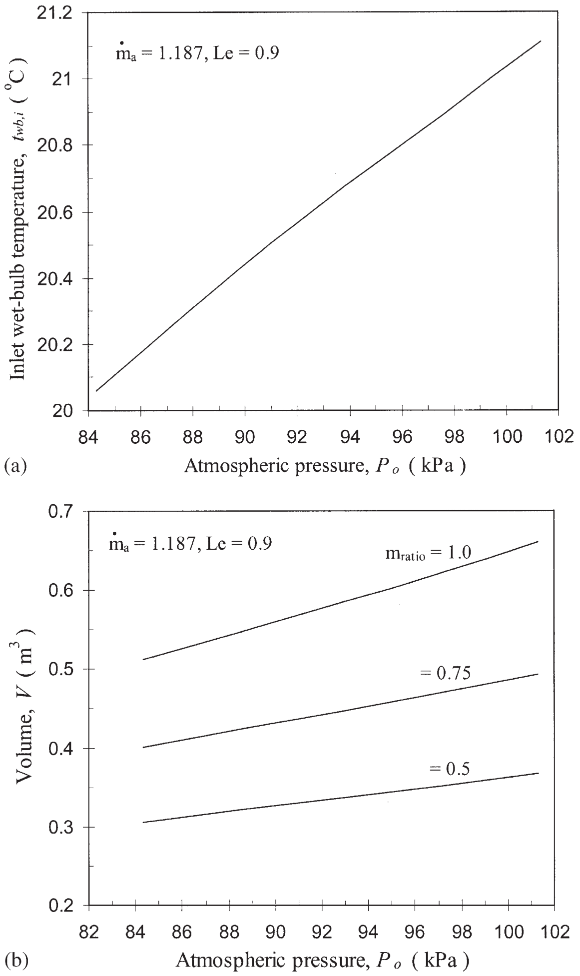

Fig. 6. Variation of moist air inlet wet bulb temperature (a) and volume of the tower (b) versus atmospheric pressure, P_o.

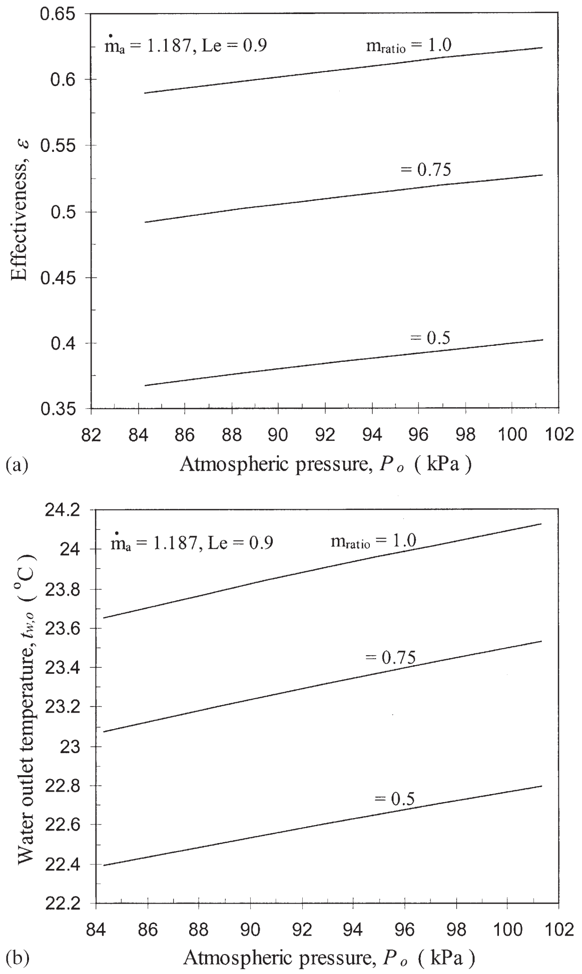

Fig. 7. Variation of tower effectiveness (a) and water outlet temperature (b) versus atmospheric pressure, P_o.

should be noted that Khan and Zubair [7] have demonstrated that the variations of wet bulb of moist air have a significant effect on the cooling tower performance. Fig. 6(a) shows that the moist air wet bulb temperature decreases by 1.0 °C when the atmospheric pressure P_o decreases by 17 kPa, whereas the plot of tower volume versus the atmospheric pressure is presented in Fig. 6(b). These plots are drawn for the following set of input data that is considered in Simpson and Sherwood [12]: $t_{db,i} = 29.0$ °C, $t_{wb,i} = 21.1$ °C, $t_{w,i} = 28.7$ °C, $t_{w,o} = 24.22$ °C, Le $= 0.9$. The figure shows that for achieving the same water outlet temperature, the volume of the tower can be reduced by 0.15 m^3 when $m_{ratio} = 1.0$. However, this decrease in volume is less as the value of mass flow rate ratio decreases.

The variation of effectiveness and water outlet temperature for rating calculations of the cooling tower is shown in Fig. 7 for the above input data. The plots are drawn for different values of mass flow ratios, m_{ratio}. The figure shows that when the atmospheric pressure P_o decreases by 17 kPa the effectiveness of the tower (Fig. 7(a)) reduces by 0.05 units for $m_{ratio} = 0.5$. This decrease in effectiveness is less for higher mass flow rate ratios. On the other hand, the water outlet temperature reduces by 0.5 °C for $m_{ratio} = 1.0$ (Fig. 7(b)); however, this decrease is less for lower mass flow rate ratios.

6. Sensitivity analysis

Any independent variable X can be represented as

$$X = \bar{X} \pm U_X \tag{17}$$

where $\bar{X}$ denotes its nominal value and U_X its uncertainty about the nominal value. The $\pm U_X$ interval is defined as the band within which the true value of the variable X can be expected to lie with a certain level of confidence (typically 95%). On the other hand if a function $Y(X)$ represents an output parameter, then the uncertainty in Y due to an

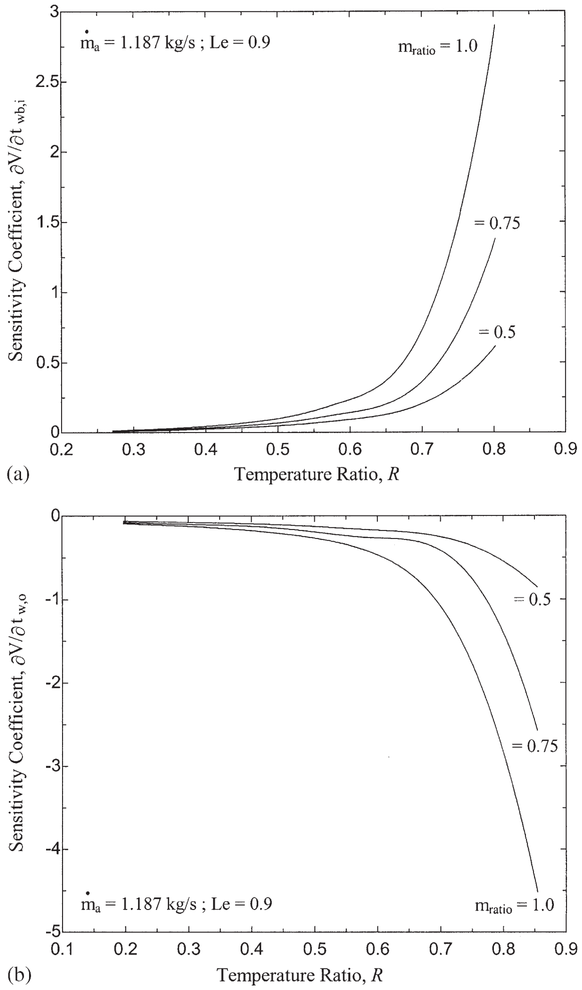

Fig. 8. Variation of volume sensitivity coefficient with respect to (a) air inlet wet-bulb temperature and (b) water outlet temperature versus R.

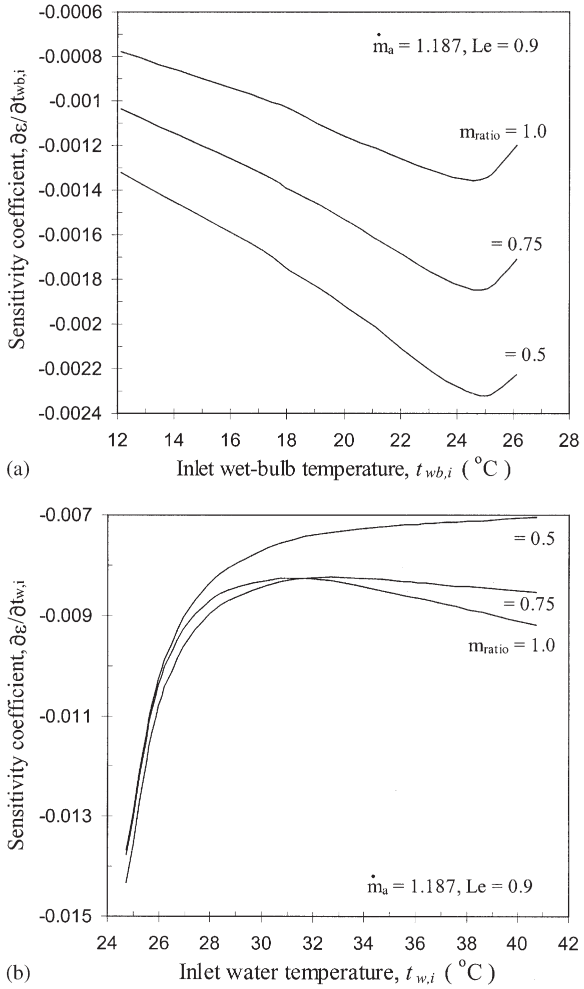

Fig. 9. Variation of tower effectiveness sensitivity coefficient with respect to (a) air inlet wet-bulb temperature, and (b) inlet water temperature.

uncertainty in X is expressed in a differential form [14]

$$U_Y = \frac{\mathrm{d}Y}{\mathrm{d}X} U_X \tag{18}$$

For a multivariable function $\boldsymbol{Y} = \boldsymbol{Y}(\boldsymbol{X}_1, \boldsymbol{X}_2, \boldsymbol{X}_3, \ldots, \boldsymbol{X}_N)$, the uncertainty in Y due to uncertainties in the independent variables is given by the root sum square product of the individual uncertainties computed to first order accuracy as

$$U_Y = \left[\sum_{i=1}^{N} \left(\frac{\partial Y}{\partial X_i} U_{X_i} \right)^2 \right]^{1/2} \tag{19}$$

Physically, each partial derivative in the above equation represents the sensitivity of the parameter Y to small changes in the independent variable X_i. It is important to note that the partial derivatives are typically defined as the sensitivity coefficients.

The computer model of the cooling tower discussed earlier in this paper is used for studying the sensitivity analysis of the cooling tower. Khan and Zubair [7] reported that, for design calculations of the cooling tower, the air inlet wet bulb temperature and water outlet temperature are the two most important input parameters influencing the performance of cooling towers. Fig. 8(a) and (b) shows the plot of volume sensitivity with respect to the inlet air wet bulb temperature and water outlet temperature, respectively. The plot is drawn for the design calculations of the cooling tower data that is used earlier in producing Fig. 6. The curves are drawn for different values of the non-dimensional temperature ratio (R) defined as

$$R = \frac{t_{\mathrm{w,i}} - t_{\mathrm{w,o}}}{t_{\mathrm{w,i}} - t_{\mathrm{wb,i}}} \tag{20}$$

Fig. 8(a) is a plot between volume sensitivity coefficient

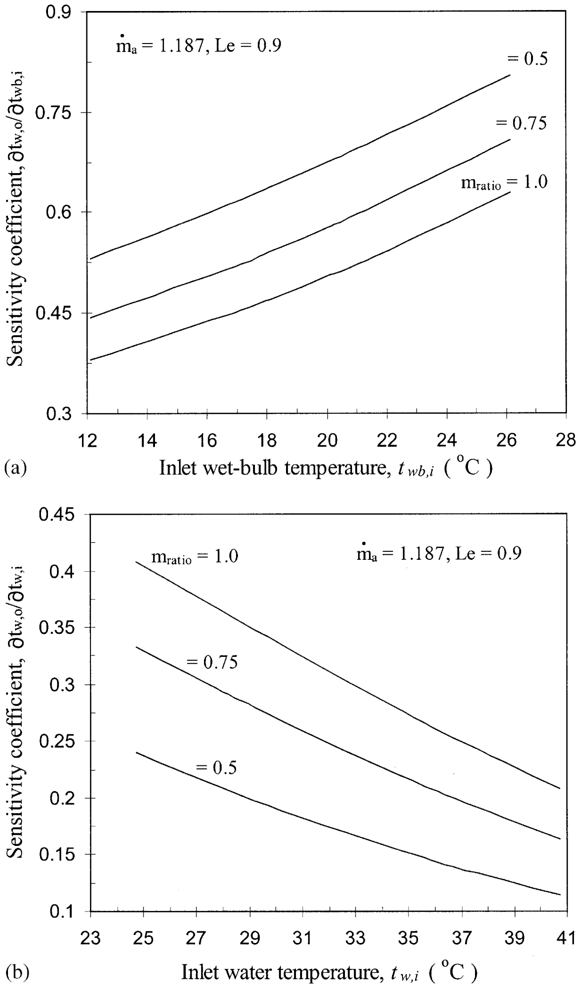

Fig. 10. Variation of water outlet temperature sensitivity coefficient with respect to (a) air inlet wet-bulb temperature, and (b) inlet water temperature.

($\partial V/\partial t_{wb,i}$) and non-dimensional temperature ratio R, for different values of mass flow rate ratio ($\dot{m}_w/\dot{m}_a$). It should be noted that, in this plot, $t_{wb,i}$ is varied from 12.11 to 23.11 °C. The figure shows that as the value of $t_{wb,i}$ increases, the sensitivity of volume with respect to $t_{wb,i}$ also increases. This increase in sensitivity with R is higher for large mass flow rate ratios. Fig. 8(b) is a plot between volume sensitivity coefficient ($\partial \boldsymbol{V}/\partial \boldsymbol{t}_{w,o}$) and R. In these plots, $t_{w,o}$ was varied from 22.22 to 27.22 °C. We find that as the value of $t_{w,o}$ increases, the sensitivity of volume with respect to $t_{w,o}$ increases though it is in the negative direction, indicating that the tower volume increases with $t_{w,o}$. This increase in sensitivity coefficient is higher for large mass flow rate ratios. As expected, the sensitivity coefficient with respect to $t_{w,o}$ is greater than $t_{wb,i}$ but they are in opposite directions.

Fig. 9(a) is a plot between effectiveness sensitivity coefficient ($\partial \varepsilon/\partial t_{wb,i}$) and inlet air wet bulb temperature ($t_{wb,i}$), for different values of mass flow rate ratio ($\dot{m}_w/\dot{m}_a$). These plots are drawn for rating calculations of the cooling tower data mentioned earlier in Fig. 7 at standard atmospheric pressure. It shows that as $t_{wb,i}$ increases, the sensitivity of effectiveness with respect to $t_{wb,i}$ increases; however the values are negative. This increase in sensitivity with $t_{wb,i}$ is lower for large mass flow rate ratios; though, it is noted that the slope reaches a maximum around 25 °C in each case. Fig. 9(b) is a plot between effectiveness sensitivity coefficient ($\partial \boldsymbol{\varepsilon}/\partial \boldsymbol{t}_{w,i}$) and water inlet temperature ($t_{w,i}$). We find that as water inlet temperature increases, the sensitivity of effectiveness with respect to $t_{w,i}$ decreases. For the temperature range investigated, it is important to note that the sensitivity coefficient with respect to $t_{w,i}$ decreases to a certain minimum value and then increases with the inlet water temperature, particularly for high mass flow rate ratios.

Fig. 10(a) is a plot between the water outlet temperature sensitivity coefficient ($\partial \boldsymbol{t}_{w,o}/\partial \boldsymbol{t}_{wb,i}$) and inlet air wet bulb temperature ($t_{wb,i}$), for different values of mass flow rate ratio ($\dot{m}_w/\dot{m}_a$). This plot is also drawn for the rating calculations of the cooling tower data mentioned earlier in Fig. 7 at standard atmospheric pressure. It shows that as $t_{wb,i}$ increases, the sensitivity of the water outlet temperature with respect to $t_{wb,i}$ also increases. We find that it is lower for large mass flow rate ratios. Fig. 10(b) is a plot between water outlet temperature sensitivity coefficient ($\partial \boldsymbol{t}_{w,o}/\partial \boldsymbol{t}_{w,i}$) and water inlet temperature ($t_{w,i}$). The data shows that the sensitivity coefficients decrease as water inlet temperature increases, which are higher for large mass flow rate ratios.

7. Concluding remarks

A cooling tower model is investigated by using Engineering Equation Solver (EES) program, which is validated with the experimental data (within ±0.26%) reported in the literature. It is demonstrated through a case study that the model developed is more accurate than earlier models reported in the literature. An asymptotic fouling model in conjunction with the numerical model of the counter flow wet cooling tower has been used to study the risk based performance characteristics of cooling towers, including the effect of fouling on fill performance index. It is demonstrated that there is about 6.0% decrease in effectiveness and about 1.2% increase in water outlet temperature for the given fouling model. In addition an example problem is presented wherein it is demonstrated that volume of the tower has to be increased in order to achieve the targeted thermal performance under fouled conditions.

The cooling tower model is also used to study sensitivity analysis of various important response variables of the tower. For example, in a design calculation of cooling

towers, the sensitivity of its volume with respect to inlet air wet bulb temperature and water outlet temperature is investigated. It is demonstrated that the sensitivity of tower volume with respect to $t_{w,o}$ is greater than $t_{wb,i}$. In a rating calculation of cooling towers, the sensitivity of its effectiveness and water outlet temperature with respect to inlet air wet bulb temperature and water inlet temperature is investigated for different mass flow rate ratios. Furthermore, the effect of atmospheric pressure on the tower performance is also studied to highlight the significance of atmospheric pressure.

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